$$D_n=\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & C_2^1 & C_3^1 & \cdots & C_{n-1}^1 & C_n^1 \\ 1 & C_3^2 & C_4^2 & \cdots & C_n^2 & C_{n+1}^2 \\ \vdots & \vdots & & & & \\ 1 & C_n^{n-1} & C_{n+1}^{n-1} & \cdots & C_{2n-3}^{n-1} & C_{2n-2}^{n-1} \end{vmatrix}_{n\times n}.$$

$$a_{ij}=C_{i+j-2}^{i-1}$$

$$\xlongequal[\substack{c_{n-1}-c_{n-2}\\ \vdots \\ c_2-c_1}]{c_n-c_{n-1}}\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & C_2^1 & C_3^1 & \cdots & C_{n-1}^1 & C_n^1 \\ \vdots & \vdots & \vdots & & \vdots & \\ 1 & C_{n-1}^{n-2} & C_n^{n-2} & \cdots & C_{2n-4}^{n-2} & C_{2n-3}^{n-2} \end{vmatrix}.$$

$$=\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ C_2^1 & C_3^1 & \cdots & C_{n-1}^1 & C_n^1 \\ \vdots & \vdots & & \vdots & \vdots \\ C_{n-1}^{n-2} & C_n^{n-2} & \cdots & C_{2n-4}^{n-2} & C_{2n-3}^{n-2} \end{vmatrix}_{(n-1)\times(n-1)}.$$

$$\xlongequal[\substack{c_{n-2}-c_{n-3}\\ \vdots \\ c_2-c_1}]{c_{n-1}-c_{n-2}}\begin{vmatrix} 1 & 0 & \cdots & 0 & 0 \\ C_2^1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & & & \\ C_{n-1}^{n-2} & C_{n-1}^{n-3} & \cdots & C_{2n-5}^{n-3} & C_{2n-4}^{n-3} \end{vmatrix}$$

$$\xlongequal[\substack{r_{n-2}-r_{n-3}\\ \vdots \\ r_2-r_1}]{r_{n-1}-r_{n-2}}\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & C_2^1 & \cdots & C_{n-2}^1 & C_{n-1}^1 \\ \vdots & \vdots & & \vdots & \vdots \\ 1 & C_{n-1}^{n-2} & \cdots & C_{2n-5}^{n-2} & C_{2n-6}^{n-2} \end{vmatrix}_{(n-1)\times(n-1)}$$

$$=\cdots=1.$$

红色为课上解法，化为低一阶的行列式恰好为右上角部分的行列式，即依次为矩阵中红线框住的本分。

$$=D_{n-1}=\cdots=D_2=1.$$